

Section 2.1 – Quadratic Functions

Definition of a Quadratic Function

- Let a , b , and c be real numbers with $a \neq 0$.
The function $f(x) = ax^2 + bx + c$ is called a QUADRATIC function in standard form.
- " a " is called the leading coefficient
 - If the leading coefficient is positive, the parabola opens upward (like a smile)
 - If the leading coefficient is negative, the parabola opens downward (like a frown)
- All parabolas are symmetric with respect to a line called the axis of symmetry
(Or simply the *axis* of the parabola)
- The point where the axis intersects the parabola is the vertex of the parabola.

Vertex Form of a Quadratic Function

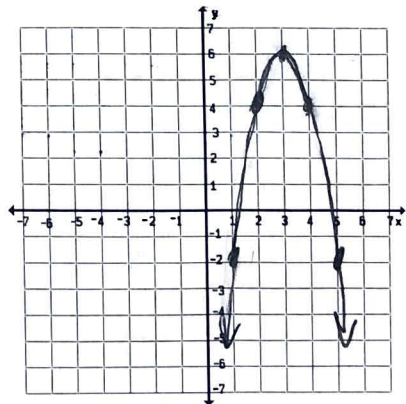
- The quadratic function $f(x) = a(x-h)^2 + k$, $a \neq 0$ is in vertex form.
- The graph of f is a parabola whose axis of symmetry is the vertical line $x = h$ and whose vertex is (h, k) .

Examples

- 1) Sketch a graph of the parabola and identify the vertex and axis of symmetry of the parabola.

a. $f(x) = -2(x-3)^2 + 6$

vertex $(3, 6)$
axis $x = 3$
vertical stretch by factor
of 2
reflect over x -axis



Section 2.1 – Quadratic Functions

If a quadratic function is in standard form, it is easier to find the vertex of the graph like this:

The vertex of $f(x) = ax^2 + bx + c$ is (h, k) , where $h = \frac{-b}{2a}$ and $k = f(h)$

2) Sketch a graph of the parabola and identify the vertex and x-intercepts of the parabola.

a) $g(x) = x^2 + 2x + 1$

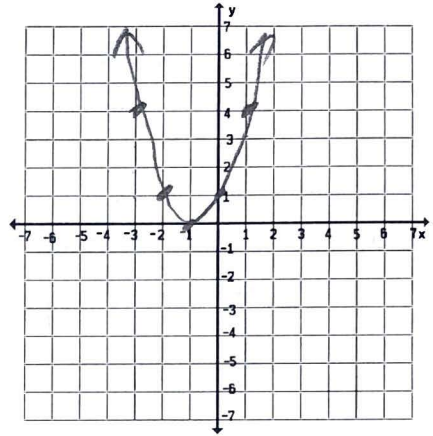
$$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

$$k = g(-1) = (-1)^2 + 2(-1) + 1 = 0$$

$$\text{vertex} = (h, k) = (-1, 0)$$

$$\text{axis } x = -1$$

$$x\text{-intercept} = \text{vertex} = (-1, 0)$$



b) $f(x) = x^2 + 2x - 30$

$$h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$$

$$k = f(-1) = (-1)^2 + 2(-1) - 30 = -31$$

$$\text{vertex} = (h, k) = (-1, -31)$$

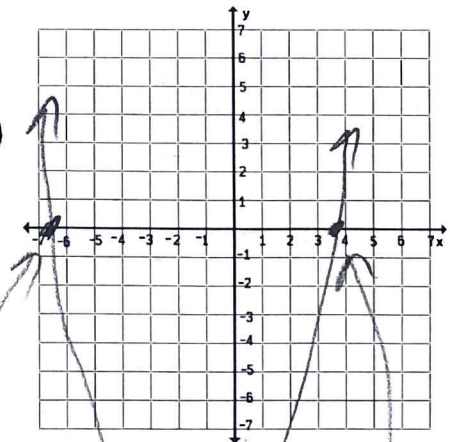
x-intercepts:

$$0 = x^2 + 2x - 30$$

$$\Rightarrow \frac{-2 \pm \sqrt{2^2 - 4(1)(-30)}}{2(1)}$$

$$\Rightarrow -1 \pm \sqrt{31}$$

$$x\text{-intercepts are } (-1 + \sqrt{31}, 0) \text{ and } (-1 - \sqrt{31}, 0)$$



Section 2.1 – Quadratic Functions

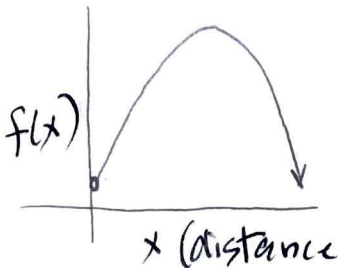
3) The Maximum Height of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45 degrees with respect to the ground. The path of the baseball is given by the function

$$f(x) = -0.0032x^2 + x + 3$$

where $f(x)$ is the height of the baseball in feet, and x is the horizontal distance from home plate in feet.

What is the maximum height reached by the baseball?



max is vertex!

$$\frac{-b}{2a} = \frac{-1}{2(-.0032)} = \frac{625}{4} = 156.25$$

$$f(156.25) = 81.125 \text{ ft}$$

4) Finding the Vertex of a Parabola by completing the square! (instead of using $h = \frac{-b}{2a}$)

(note: this method is easiest to use when $a = 1$)

a) $f(x) = x^2 + 8x + 11$

Vertex: $(-4, -5)$

$$= (x^2 + 8x + 16) + 11 - 16$$

$$= (x + 4)^2 - 5$$

b) $f(x) = x^2 - 14x + 9$

Vertex: $(7, -40)$

$$= (x^2 - 14x + 49) + 9 - 49$$

$$= (x - 7)^2 - 40$$

Section 2.1 – Quadratic Functions

5) Writing the Equation of a Parabola

- a) Write the vertex form of the equation of the parabola whose vertex is (1,2) and that passes through the point (0,0).
Sketch a graph too!

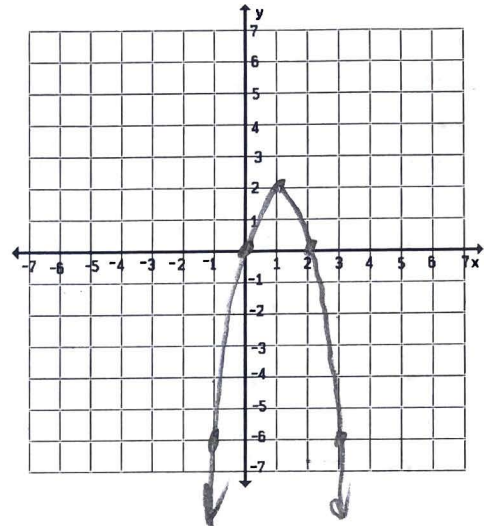
$$f(x) = a(x-1)^2 + 2$$

Plug in (0,0) ↗

$$0 = a(0-1)^2 + 2$$
$$-2 = a$$

↓

$$f(x) = -2(x-1)^2 + 2$$



- b) Write the vertex form of the equation of the parabola whose vertex is (4,-1) and that passes through the point (2,3).
Sketch a graph too!

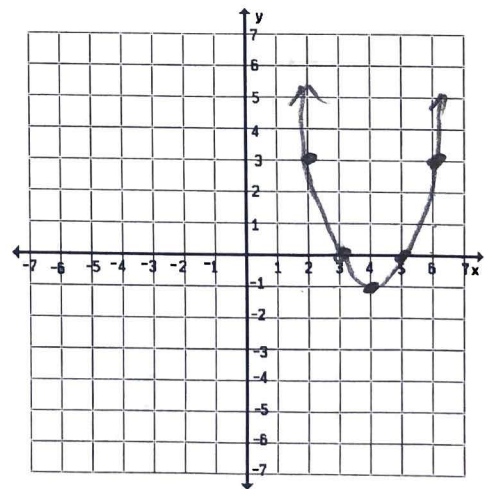
$$f(x) = a(x-4)^2 - 1$$

plug in (2,3)

$$3 = a(2-4)^2 - 1$$
$$1 = a$$

↓

$$f(x) = (x-4)^2 - 1$$



Section 2.1 – Quadratic Functions

- 6) A study was done to compare the speed x (in miles per hour) with the mileage y (in miles per gallon) of an automobile. The results are shown in the table.

Speed, x	Mileage, y
15	22.3
20	25.5
25	27.5
30	29.0
35	28.8
40	30.0
45	29.9
50	30.2
55	30.4
60	28.8
65	27.4
70	25.3
75	23.3

- a) Use your graphing calculator to create a scatter plot of the data (STAT- EDIT—enter the data)
- b) Use the regression feature of the calculator to find a quadratic model for the data (STAT- CALC- QuadReg (VARS - Y-VARS - Y_1) - ENTER)

$$y = -.0081998002x^2 + .746113886x + 13.46863137$$

- c) Enter the equation you just found into the "y =" so that it graphs the regression on top of the scatter plot. (or put y_1 into "store RegEQ")
- d) Estimate the speed for which the mileage is the **greatest**.

$$45.495857 \text{ mph}$$

$$(30.441177 \text{ mi})$$

$$\text{or } \frac{-b}{2a} = \frac{-.746113886}{2(-.0081998002)}$$

$$= 45.4959$$

Homework Day 1: p.134 #1-8, 9, 13, 18, 24, 25, 37, 40, 44, 45, 53, 56, 65 (where it says "standard form" use "vertex form")

Homework Day 1 ½: p.136, #77,78,79